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Your Roll No.....

5192F

**B.Sc. (Hons.)/(Physical Science)/**

**Mathematical Science/II Sem. B**

Paper MAPT—202

MATHEMATICS—II (Calculus and Geometry)

(Admission of 2010 and onwards)

*Time : 3 Hours*

*Maximum Marks : 75*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*All the questions are compulsory.*

Attempt any *two* parts from each question.

Marks of each question are indicated.

1. (a) (i) Let the function be defined as :

$$f(x) = \frac{x-2}{|x|-2}$$

P.T.O.

Find the value of  $x$  (if any) at which  $f$  is not continuous and determine whether each such value is a removable discontinuity.

(ii) Use  $(\epsilon, \delta)$  definition to prove that 6

$$\lim_{x \rightarrow 3} x^2 = 9.$$

(b) Let  $f$  be defined on  $\mathbb{R}$  by setting :

$$f(x) = 1 \text{ when } x \text{ is irrational}$$

$$f(x) = -1 \text{ when } x \text{ is rational}$$

Show that  $f$  is discontinuous at every point of  $\mathbb{R}$ . 6

(c) Define uniform continuity. Show that the function  $f(x) = \sqrt{x}$  for  $x \in [0, 1]$  is uniformly continuous. 6

2. (a) Discuss the derivability of the function : 6

$$f(x) = 2x - 3 \quad 0 \leq x \leq 2$$

$$= x^2 - 3 \quad 2 \leq x \leq 4$$

at  $x = 2, 4$ .

(b) State and prove Rolle's theorem and give its geometrical interpretation. 6

(c) Find all the asymptotes of the Cartesian curve :

$$x^2 y^2 (x^2 - y^2)^2 = (x^2 + y^2)^3. \quad 6$$

3. (a) (i) Find the points of inflexion for the curve :

$$a^2 y^2 = x^2 (a^2 - x^2).$$

(ii) Find the interval on which the function : 6

$$f(x) = 4 - 3x - x^2$$

is concave up or concave down.

- (b) Find the position and nature of the double points of the curve :

$$y^2 = (x - a)^2 (x - b).$$

6

- (c) Trace the curve :

$$r = a \sin 3\theta, \quad a > 0.$$

6

4. (a) Trace the curve :

$$y(a^2 + x^2) = a^2 x.$$

7

- (b) Obtain reduction formula for

$$I_n = \int x^n (a - x)^{1/2} dx.$$

Prove that :

$$(2n + 3) I_n = 2a_n I_{n-1} - 2x^n (a - x)^{3/2}.$$

7

- (c) Find the whole area of the curve :

$$x^2(x^2 + y^2) = a^2(x^2 - y^2).$$

7

5. (a) Find the volume of the solid generated by the revolution of the cycloid  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$  about  $x$ -axis.

6

- (b) Describe the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

6

- (c) (i) Find an equation for a hyperbola whose vertices  $(\pm 1, 0)$  asymptotes  $y = \pm 2x$  are given.

- (ii) Discuss reflection property of parabola together

with figure.

6

6. (a) Let an  $x' y'$  co-ordinate system be obtained by rotating an  $xy$  co-ordinate system through an angle of  $\theta = 30^\circ$ .

Find an equation of the curve :

$$2x^2 + 2\sqrt{3} xy = 3 \text{ in } x' y'$$

co-ordinate.

6½

- (b) (i) Find :

$$\nabla \times (\nabla \times \vec{F})$$

for

$$\vec{F}(x, y, z) = y^2 x \hat{i} - 3yz \hat{j} + xy \hat{k}.$$

- (ii) Prove that :

$$\text{div}(\text{curl } \vec{F}) = 0$$

where

$$\vec{F} = \vec{F}(x, y, z).$$

6½

- (c) (i) Sketch the graph

$$x^2 + z^2 = 1$$

in 3-dimensional space.

- (ii) Show that :

$$\frac{d}{dt}[r(t) \times r'(t)] = r(t) \times r''(t). \quad 6\frac{1}{2}$$